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PROBLEM SOLVING USING THE SINE AND COSINE RULES

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When using trigonometry to solve problems, you should draw a diagram of the situation. The diagram should be reasonably accurate, and all important information should be clearly marked on it.

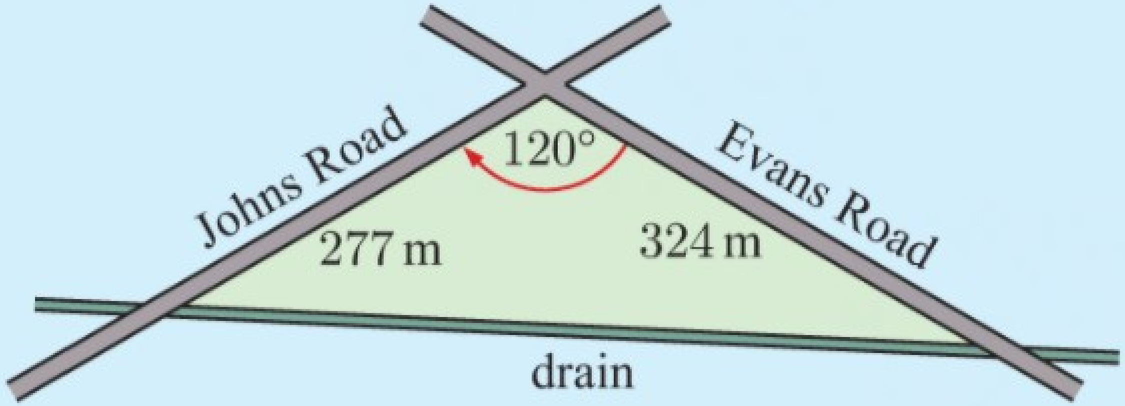
Example 18

Self Tutor

A triangular property is bounded by two roads and a long, straight drain. Find:

a the area of the property in hectares

b the length of the drain boundary.



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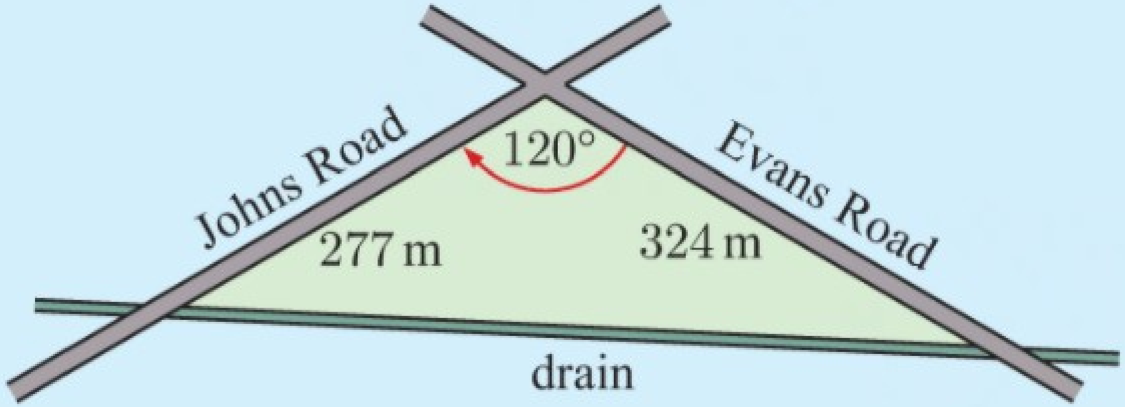
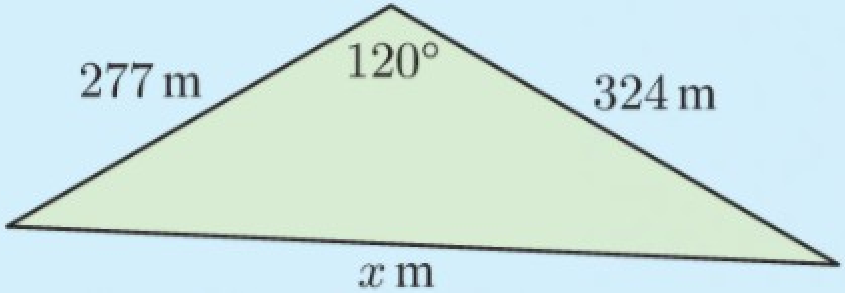


a Area $= \frac{1}{2} \times 277 \times 324 \times \sin 120°$

$\approx 38\,862 \text{ m}^2$

$\approx 3.89 \text{ ha} \qquad \{1 \text{ ha} = 10\,000 \text{ m}^2\}$



b Let the drain boundary be x m long.

$\therefore \; x^2 = 277^2 + 324^2 - 2 \times 277 \times 324 \times \cos 120° \quad$ {cosine rule}

$\therefore \; x = \sqrt{277^2 + 324^2 - 2 \times 277 \times 324 \times \cos 120°}$

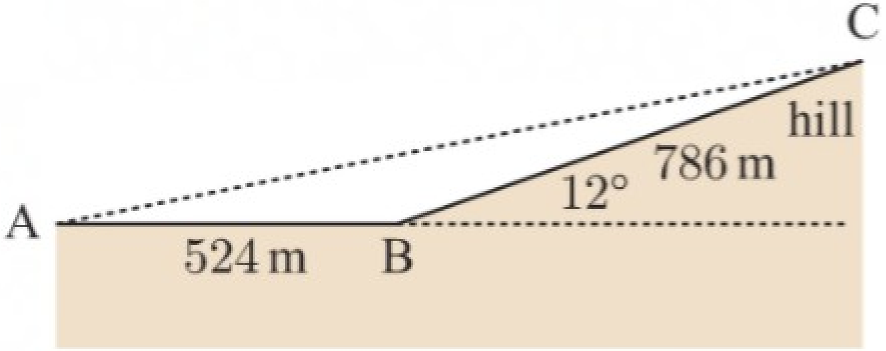
$\therefore \; x \approx 521$

The drain boundary is approximately 521 m long.

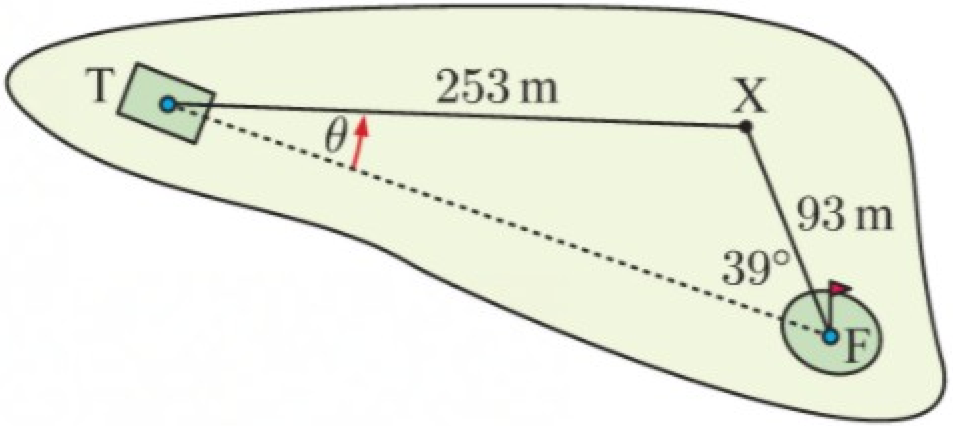
EXERCISE 12I

1 Two farm houses A and B are 10.3 km apart. A third farm house C is located such that $B\widehat{A}C = 83°$ and $A\widehat{B}C = 59°$. How far is C from A?

2 A roadway is horizontal from A to B, then rises up a 12° incline from B to C.
How far is it directly from A to C?



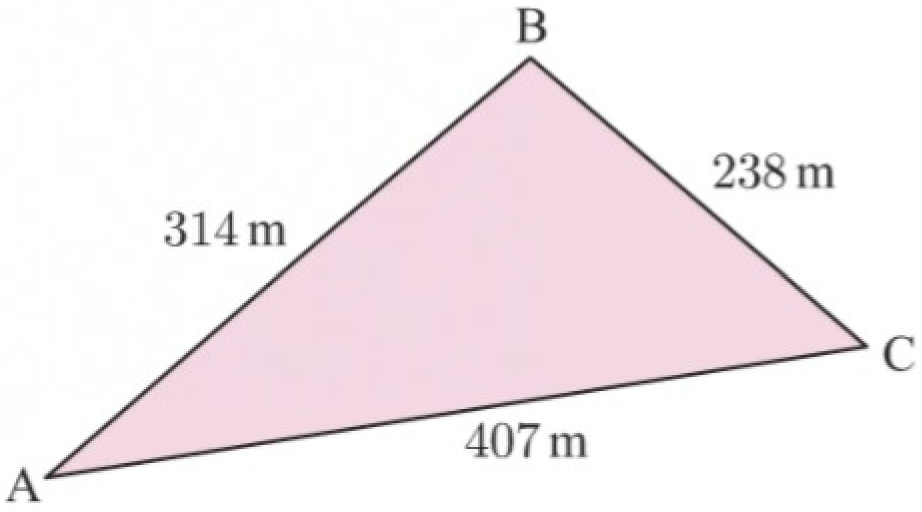
3 Sharon drives a golf ball 253 m from the tee T to point X on the fairway. X is 93 m from the flag, and $X\widehat{F}T$ is 39°.
Find the angle θ that Sharon's drive was off line.



4 Hazel's property is triangular with the dimensions shown.

a Find the measure of the angle at A, rounded to 2 decimal places.

b Find the area of Hazel's property, to the nearest hectare.



Example 19 **Self Tutor**

A ship sails 58 km on the bearing 072°. Once it has passed a reef, it turns and sails 41 km on the bearing 158°. How far is the ship from its starting point?

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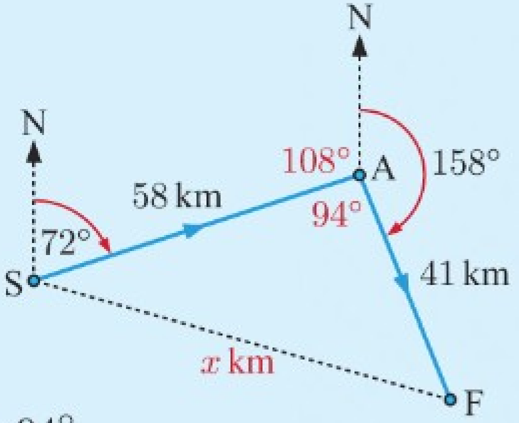

We suppose the ship starts at S, sails to A, then changes direction and sails to F.

$$\widehat{SAN} = 180° - 72° = 108° \quad \{\text{co-interior angles}\}$$

$$\therefore \quad \widehat{SAF} = 360° - 158° - 108° = 94° \quad \{\text{angles at a point}\}$$

Let $SF = x$ km.

Using the cosine rule, $x^2 = 58^2 + 41^2 - 2 \times 58 \times 41 \times \cos 94°$

$$\therefore \quad x = \sqrt{58^2 + 41^2 - 2 \times 58 \times 41 \times \cos 94°}$$

$$\therefore \quad x \approx 73.3$$

The ship is about 73.3 km from its starting point.

5 Fred walks 83 m in the direction 111°, and then 78 m in the direction 214°. How far is Fred from his starting point?

6 A boat travels 13 km in the direction 138°, and then a further 11 km in the direction 067°. Find the distance and bearing of the boat from its starting point.

7 Mount X is 9 km from Mount Y, on a bearing 146°. Mount Z is 14 km from Mount X, and on a bearing 072° from Mount Y. Find the bearing of X from Z.

8 X is 20 km north of Y. A mobile telephone mast M is to be placed 15 km from Y so the bearing of M from X is 140°.

- **a** Draw a sketch to show the two possible positions where the mast could be placed.
- **b** Calculate the distance of each of these positions from X.

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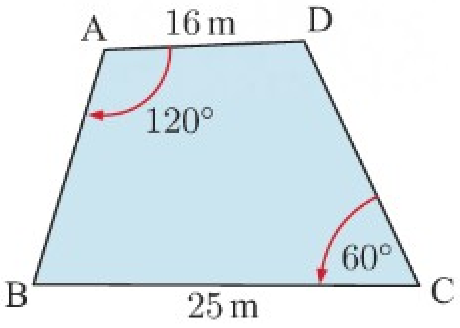
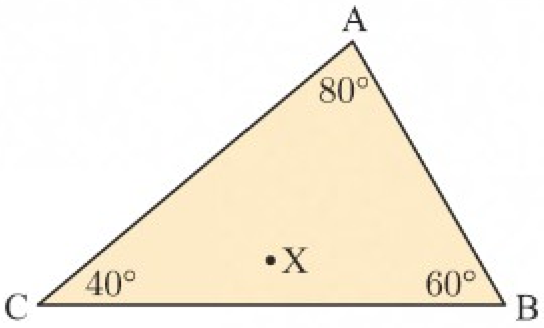
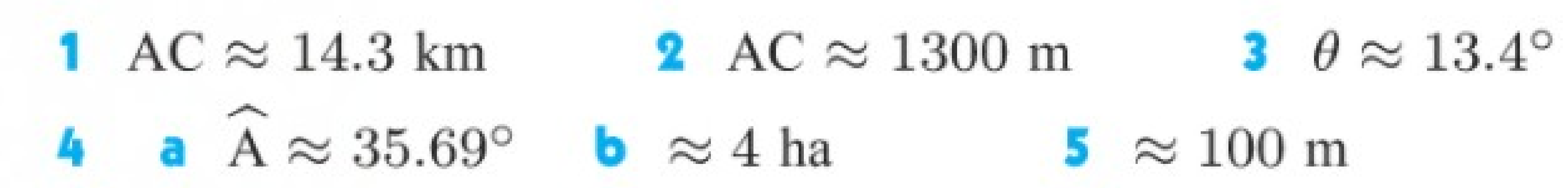


The quadrilateral ABCD represents David's garden plot. AD = 16 m, BC = 25 m, and [DC] is 5 m longer than [AB]. A fence runs around the entire boundary of the plot. How long is the fence?

10 In the given triangle, X is 5 m from each of the vertices. Find the length of each side of the triangle.



11 The angles of a triangle measure 104°, 51°, and 25°. The perimeter of the triangle is 10 m. Find, rounded to 2 decimal places, the length of each side of the triangle.



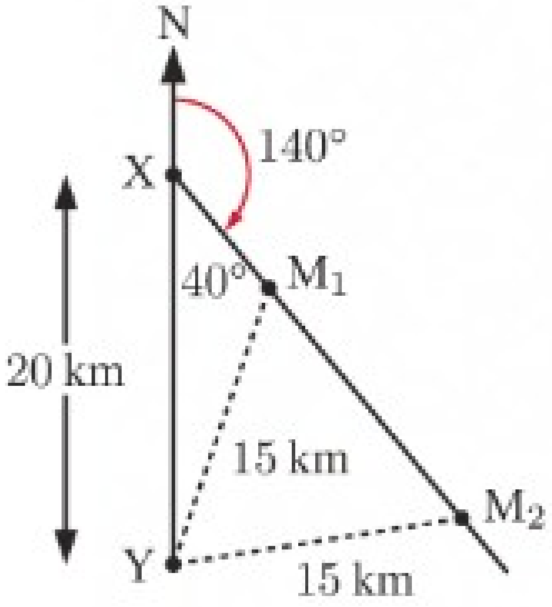
1 AC $\approx$ 14.3 km **2** AC $\approx$ 1300 m **3** $\theta \approx 13.4°$

4 **a** $\widehat{\mathrm{A}} \approx 35.69°$ **b** $\approx$ 4 ha **5** $\approx$ 100 m

6 19.6 km in direction 106°

7 $\approx 214°$

8 **a**



b $XM_1 \approx 7.59$ km
$XM_2 \approx 23.0$ km

9 ≈ 63.4 m

10 $AB \approx 6.43$ m,
$BC \approx 9.85$ m,
$AC \approx 8.66$ m

11 ≈ 3.58 m,
≈ 1.95 m,
≈ 4.47 m